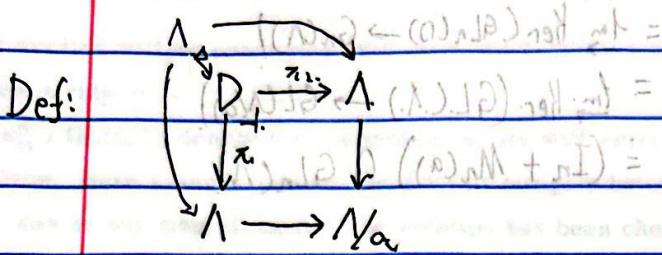


Lecture 5: 10/6.

§4.  $K_1(a) \rightarrow K_1(\Lambda) \rightarrow K_1(\Lambda/a) \rightarrow K_0(a) \rightarrow K_0(\Lambda) \rightarrow K_0(\Lambda/a)$



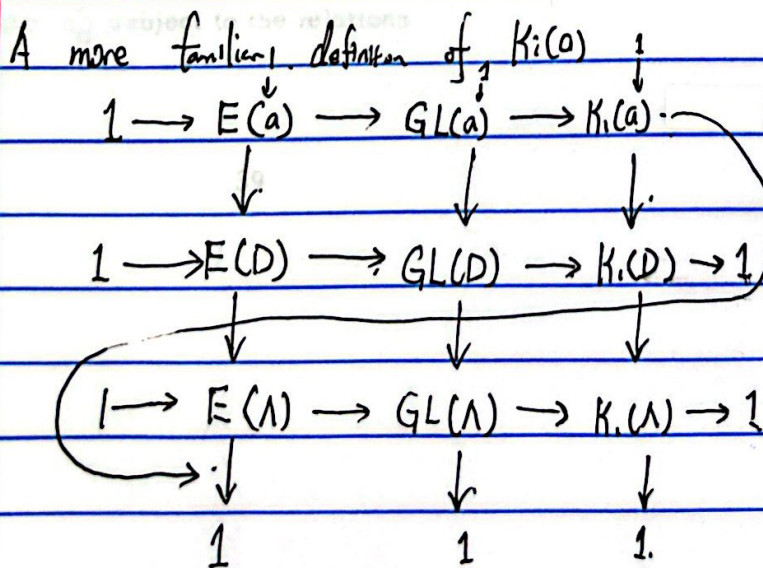
$D$  is called the double of  $\Lambda$  along  $a$ .

Remark: By Functoriality of  $K_0$  &  $K_1$ , the maps  $K_i(D) \xrightarrow{\pi_1} K_i(\Lambda) \xrightarrow{\pi_2} K_i(\Lambda/a)$  are split. epis

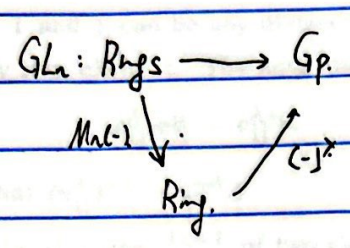
$0 \rightarrow K_i(a) \rightarrow K_i(D) \rightarrow K_i(\Lambda) \rightarrow 0$  is split exact.

Plugging into the MV Sequence in §3

$K_1(a) \oplus K_1(\Lambda) \rightarrow K_1(\Lambda) \oplus K_1(\Lambda) \rightarrow K_1(\Lambda/a) \oplus 0 \rightarrow K_0(a) \oplus K_0(\Lambda) \rightarrow K_0(a) \oplus K_0(\Lambda) \rightarrow K_0(\Lambda/a) \oplus K_0(\Lambda) \rightarrow K_0(\Lambda/a) \oplus 0$



$$\begin{aligned}
 GL(a) = \text{Ker}(GL(D)) &\longrightarrow GL(V) = \text{Ker}(\varinjlim GL_n(D)) \longrightarrow \varinjlim GL(V) \\
 &= \varinjlim \text{Ker}(GL_n(D) \rightarrow GL_n(V)) \\
 &= \varinjlim \text{Ker}(GL_n(1) \rightarrow GL_n(V)) \\
 &= (I_n + M_n(a)) \cap GL_n(V)
 \end{aligned}$$



By functoriality of  $K_0$  &  $K_1$ , the maps  $K_0(D) \rightarrow K_0(V)$  and  $K_1(D) \rightarrow K_1(V)$  are

$$0 \longrightarrow K_1(D) \longrightarrow K_1(V) \longrightarrow 0$$

$$K_0(D) \oplus K_0(V) \longrightarrow K_0(V) \oplus K_0(V) \longrightarrow K_0(V) \oplus K_0(V) \longrightarrow 0$$

