# ORAL QUALIFYING EXAM SYLLABUS

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## MAJOR AREA: ALGEBRAIC TOPOLOGY

**Topological Hochschild homology** Mona Merling (advisor)

*I will give a presentation to the committee and be prepared to answer questions* Nir Gadish

1. Basics in topological Hochschild homology
   - Motivation: Dundas-Goodwillie-McCarthy theorem.
   - Basic Hochschild homology, HKR theorem, Hochschild complex, cyclic homology.
   - Ring spectra, $E_\infty$-structures.
   - Definition of topological Hochschild homology of ring spectra, properties (change-of-basis formula and universal property).
   - Examples: THH of Thom spectra and spherical group rings $\mathbb{S}[z]$.
2. Bökstedt periodicity and even filtrations
   - Hopkins-Mahowald theorem, computation of $\mathrm{THH}(\mathbb{F}_p)$.
   - Brief idea of Dyer-Lashof operations, proof idea of Hopkins-Mahowald theorem, Bökstedt periodicity: $\pi_* \mathrm{THH}(\mathbb{F}_p) = \mathbb{F}_p[x]$ for some explicitly defined $x$ with $|x| = 2$.
   - Filtered objects and associated graded objects, evenly faithfully flat (eff) maps, even filtrations.
   - Example: $\mathrm{THH}(\mathbb{S}[t]) \to \mathbb{S}[t]$ is an eff map.
   - Motivic filtrations and examples, brief summary of associated spectral sequences.

**[References]** Selections mainly from

- Chapter 1-4, "Lectures on topological Hochschild homology and cyclotomic spectra" by A. Krause and T. Nikolaus,
- Talbot 2024 notes by various contributors,
- Chapter 4-7, 16 (in directory order) "Arbeitsgemeinschaft: Topological Cyclic Homology (2018)" notes by various contributors,
- Chapter 4 "A motivic filtration on the topological cyclic homology of commutative ring spectra" by J. Hahn, A. Raksit, and D. Wilson,
- "Bökstedt periodicity and quotients of DVRs" by A. Krause and T. Nikolaus,

and various research papers by T. Nikolaus and A. Krause, B. Bhatt, J. Hahn, G. Wang, R. Liu et al.

## MINOR AREA: ALGEBRAIC GEOMETRY

**Chow-Witt theory** Daniel Krashen

*I will write a master's thesis and (probably) present at the graduate geometry/topology seminar*

1. Chow theory
   - Basic background in schemes, algebraic cycles, divisors, Chow groups, basic properties of Chow groups.
   - Chern classes, pushouts and pullbacks, localization sequences, example of computations.
   - Euler classes, Chow rings, examples of computations of Chow rings.
2. Milnor-Witt K-theory
   - Milnor K-theory rings, Milnor-Witt K-theory rings.
   - Quadratic forms, symmetric bilinear forms, Witt groups, Grothendieck-Witt rings.

- Relationship between Grothendieck-Witt rings and Milnor-Witt K-theory rings. More properties.

3. Chow-Witt theory
   - Rost-Schmid complexes, Chow-Witt groups.
   - Properties of Chow-Witt groups, pushouts and pullbacks, localization sequences, relation with Chow groups, motivic cohomology.
   - Examples of computations in Chow-Witt theory, e.g. Chow-Witt groups of projective spaces.

[**References**] Selections from "Notes on Milnor-Witt K-theory" by F. Déglise, "Classifying spaces in motivic homotopy theory" by B. Totaro in PCMI 2024, "Lectures on Chow-Witt groups" by J. Fasel, and various research papers by F. Morel and V. Voevodsky.