

READING COURSE: ALGEBRAIC K-THEORY

This is a reading course on algebraic K-theory. In this course, we will cover (a certain subset of) the following topics:

Basic Topics in Classical K-theory

- Projective modules and K_0 of rings. K_0 of Dedekind domains. (Milnor Ch. 1)
- K_1 of rings. (Milnor Ch. 3)
- Mayer-Vietoris Sequence for K_0 and K_1 . (Milnor Ch. 2 and 3)
- Steinberg group and K_2 . (Milnor Ch. 5)
- Excision exact sequence for K_0, K_1 and K_2 . (Milnor Ch. 4 and 6)
- The product $K_1 \otimes K_1 \rightarrow K_2$. (Milnor Ch. 8)
- transfer homomorphisms. (Milnor Ch. 14)

Applications of Classical K-theory

- Milnor K -theory and Matsumoto's theorem on K_2 of fields. (Milnor Ch. 11, 12)
- K -theory of \mathbb{Z} , Dedekind domains and number fields. (Milnor Ch. 9, 10, 13, 16)
- Norm residue symbol, Merkurjev–Suslin theorem, Bloch-Kato Conjecture. (Milnor Ch. 15, Srinivas Ch. 8)
- Wall's finiteness obstruction, Whitehead torsion. (Lurie's lecture 2-4)

Warning: As you can see, the contents are presented topically rather than in the order of Milnor's book to make the topics clearer. It may be wise to follow the original order of the book.

Topics in Modern K-theory

- Definition of higher K-theory, Q -construction, $+$ -construction. (Weibel Ch. 4, Srinivas Ch. 2-4)
- Waldhausen category, Waldhausen S -construction. (Weibel Ch 4.)
- “Fundamental Theorems”: Localization, devissage, and resolution. (Weibel Ch. 5)
- Brown-Gersten-Quillen spectral sequence. (Weibel Chapter 5, Srinivas Ch. 5-6)
- K -theory of finite fields. Higher K -theory of fields (Weibel Ch. 4, 6)
- (Topological) cyclic homology, (Topological) Hochschild homology.

There is no fixed reference, but the following books may be helpful:

- Introduction to Algebraic K-Theory by John Milnor
- Introduction to Algebraic K-Theory, by John R. Silvester
- Algebraic K-Theory and Its Applications by Jonathan Rosenberg
- The K-book by Charles A. Weibel
- Algebraic K-Theory by V. Srinivas

The first three focus on classical algebraic K-theory, providing explicit constructions and examples. The other two books are centered on higher K-theory and employ homotopical methods.