Special Seminar: Modern Techniques in Homotopy Theory
Summer 2025

Seminar Schedule

The reading seminar will start weekly from the week of **May 26th-30th** to the week of **August 18th-22nd**. This is 13 weeks in total, and there will be 1 lecture per week. Each lecture starting from Lecture 1 is expected to be 1.5 hours long. If you do decide to give a talk, please come talk to us about the details of what to cover. The following topics are chosen tentatively and subject to change.

Lecture 0 (05/28/2025): Logistics, high-level introduction to motivic homotopy theory.

Lecture 1 (06/04/2025): A rapid review of algebraic geometry: schemes, varieties, divisors, cohomology, categories, functor of points view.

Lecture 2 (06/11/2025): A rapid tour of Higher Algebra - simplicial sets, ∞-categories, the ∞-category of spaces, stable ∞-categories, the ∞-categories of Spectra.

Lecture 3 (06/18/2025): Grothendieck topology and sites. Zariski topology, Nisnevich topology, étale topology. Sheaf over sites. Nisnevich sheaves.

Lecture 4 (06/25/2025): Introduction to unstable motivic homotopy theory: Motivic spaces. Sheafification. Algebraic K-theory is a motivic space. Motivic spheres. Eilenberg-MacLane spaces. More examples. Comparison with the classical objects. Simple calculations if applicable.

Lecture 5 (07/02/2025): Fundamental groups of $\mathbb{P}^1$. Motivic Freudenthal suspension theorem. More motivic homotopy groups.

Lecture 6 (07/09/2025)[1]:Introduction to stable motivic homotopy theory. Motivic (ring) spectra. Symmetric monoidal structure. Basic properties. Homotopy groups. Constructions of classical examples (e.g. Milnor-Witt K-theory, KGL, etc.)

Lecture 7 (07/16/2025): (Motivic) cohomology theories, Chern classes, and computations.

Lecture 8 (07/23/2025): Motivic cohomology via the Voevodsky motivic complexes. Bloch cycle complexes. Relations to Chow groups. Properties.

Lecture 9 (07/30/2025): Milnor(-Witt) K-theory. A roadmap to normal residue theorem, and Lichtenbaum-Quillen conjecture.

Lecture 10 (08/06/2025): BGL and the unstable representation theorem of Morel-Voevodsky. KGL and the stable representation theorem.

[1]Depending on how far we get in Lecture 2, the speaker for this lecture may need to introduce the ∞-category of spectra at the beginning.

Lecture 11 (08/13/2025): Algebraic cobordism. The motivic Thom spectrum MGL and its construction. Connections to formal group laws. Cohomology for MGL.

Lecture 12 (08/20/2025): Hopkins-Morel-Hoyois. Computations of slices of KGL and MGL.

Lecture 13 (08/27/2025 or later) (if applicable): More on motivic Adams spectral sequences. Algebraic slice spectral sequences. More computations, e.g. (very) effective cover of algebraic K-theory, very effective cover of hermitian K-theory, etc.